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Gaps and Gluts: Reply to Parsons

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1 Introduction

Numerous solutions have been proposed to the semantic paradoxes. Two that are frequently singled out and compared are the following. The first is that according to which paradoxical sentences are neither true nor false — as it is sometimes put, they are semantic gaps. The second is that according to which paradoxical sentences are both true and false — as it is sometimes put, they are semantic gluts (dialetheias). Calling the first of these a solution is, in fact, somewhat misleading: it is rather like calling an opening gambit a game of chess. For the solution runs into severe problems almost immediately, and so can be only the first of a series of (often ad hoc) moves made to defend the original weak opening.[1] Nonetheless, the symmetry involved in the gap and glut solutions is obvious enough to make the comparison a natural one.

1 Thus, variations on the gambit, agreeing on nothing but the first move, abound in the literature. To cite but a few: J. Barwise and J. Etchemendy, *The Liar* (Oxford: Oxford University Press 1987); L. Goldstein, '"This Statement is not True" is not True,' *Analysis* **52** (1992) 1-5; S. Kripke, 'Outline of a Theory of Truth,' *Journal of Philosophical Logic* **72** (1975) 690-716; M. Sainsbury, *Paradoxes* (Cambridge: Cambridge University Press 1988), ch. 5; and T. Smiley, 'True Contradictions, I,' *Proceedings of the Aristotelian Society, Supplementary Volume* **68** (1993) 17-33. Those familiar with the literature will be able to cite numerous others.

Such a comparison has recently been made by Terry Parsons[2] — who is well known for his own gap solution[3] — and his conclusion is an intriguing one. He argues that the two accounts are more similar than one might imagine, being subject to parallel problems, to which parallel solutions may apply. Indeed, the two theories may just be different ways of looking at one and the same situation. Parsons's paper contains many perceptive points, and I think there is much to be learned from it. However, it seems to me that ultimately his position cannot be maintained. Though there certainly are similarities between the two theories (as one would expect in any situation of duality), at crucial points the parallel breaks down, and where it does so, this is entirely to the advantage of the glut view. Or so, at least, I will argue.

II Truth and Truth-Value Gaps

In *In Contradiction*[4] I argued that there are dialetheias, true contradictions. The case is built on numerous considerations;[5] one of these is the semantic paradoxes. Let us start with the case against gap solutions presented there. One part of this is an argument against the existence of gaps. Parsons, of course, rejects this. Let us examine his reasons.

The argument against gaps depends on an account of truth which I called the Teleological Account,[6] and which, for present purposes, is not at issue. The account may simply be stated thus: truth is that which is the (institutional) aim of various cognitive activities, notably assertion. Analogously, winning is the (institutional) aim of playing competitive

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2 'True Contradictions,' *Canadian Journal of Philosophy* **20** (1990) 335-53. Unless otherwise stated, page references are to this paper. Parsons's essay is based on a talk given at the Pacific DivisionAPA in Spring 1989. This paper is an extended version of the reply given there. I am very grateful to him for written comments on an earlier version of it.

3 T. Parsons, 'Assertion, Denial, and the Liar Paradox,' *Journal of Philosophical Logic* **13** (1984) 137-52

4 G. Priest, *In Contradiction* (The Hague: Kluwer Academic Publishers 1987). I will refer to this as *IC*.

5 Arguments from the Hegel/Marx tradition are not employed, though occasionally connections are hinted at. Parsons comments that he thinks these allusions misleading (336). I think the matter is a complex one, but certainly *some* of the true contradictions of this tradition are dialetheias. See my 'Dialectic and Dialethic,' *Science and Society* **53** (1989) 388-415; and 'Was Marx a Dialetheist?' *Science and Society* **54** (1990) 468-75.

6 *IC*, ch. 4. Both the account and the observation that it appears to rule out truth value gaps are Dummett's.

games. That there are no truth value gaps follows. For anything less than meeting the aim is failure. There is no *tertium datur*. The analogy with games is again useful here. In a multiple-person game brought to a legitimate conclusion, if a person doesn't win, she fails; but we can distinguish within the category of failures according to whether some other person wins (loss) or whether no other person does (draw). In a single-person game (such as patience) brought to a legitimate conclusion, however, there is no intrinsic ground for distinguishing between two different sorts of outcome. So it is for asserting, for assertion is a single-person game.

Parsons's objection amounts to this. Granted, there is no *tertium* between succeeding in assertion and failing: failing corresponds to not being true. This is different from being false (i.e., having a true negation). So there may yet be some non-truths that are false, and some that are not. This I concede, but it seems to me to miss the force of the argument. We may still try to distinguish, within the category of failures, two subcategories. However, as the single-person game analogy shows, there is no intrinsic ground for doing this; hence any such distinction would be spurious, and so could not ground any important semantic distinction.[7]

III Truth Value Gaps and Semantic Paradoxes

The case against a gap solution to the semantic paradoxes given in *IC* does not depend wholly, or even mainly, on the rejection of the existence of truth value gaps. *IC* 1.3 concedes, for the sake of argument, that there are truth value gaps, and argues that this solution fails anyway.[8] The reasons given there are, in increasing order of importance:

(1) even if there are gaps, without reasons for supposing paradoxical sentences to be amongst them, this solution is unsatisfactory; and no satisfactory reasons have been given.

(2) gap solutions are susceptible to extended paradoxes.

(3) there are standard logical paradoxes which are derivable even assuming there to be gaps.

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7 Parsons objects to two other arguments he claims to find in the text (343, n. 7). The first of these is a quotation by Dummett. I take this merely to express the argument that we have just considered, but I do not want to enter into questions of exegesis here. The second is actually an objection to an argument for the existence of truth value gaps, rather than an argument against them.

8 I even indicate how gaps can be incorporated into dialetheic semantics (95, n. 3).

Parsons does not address the first and third of these. His comments on (2) will take up a major part of this essay, and I will come to them in a moment. All I want to do at present is emphasize the importance of (3). Paradoxes such as Berry's, where the contradiction is not arrived at via proving something of the form $\alpha \leftrightarrow \neg\alpha$, can be derived without using the Law of Excluded Middle (LEM) and, more generally, in a logic whose semantics contains gaps. (See *IC* 1.8.) Invoking truth value gaps cannot, therefore, solve them.

IV The Extended Liar and Assertion

Let us turn to (2). The Liar sentence is a sentence, α, of the form $\neg T\langle\alpha\rangle$. (I use 'T' as a truth predicate and angle brackets as a name-forming device.) By the T-schema, $T\langle\alpha\rangle$ is inter-deducible with $\neg T\langle\alpha\rangle$. Gap solutions to the paradox insist that the negation of a gap is a gap. Hence $T\langle\alpha\rangle$ may consistently be taken to be a gap (as, therefore, may α itself).

The obvious problem with this move is that anyone who endorses such a solution appears to endorse the claim that α is neither true nor false, and so, in particular, that it is not true, i.e., $\neg T\langle\alpha\rangle$. Thus they are endorsing something that is, by their own account, untrue. Hence they can express their own attitude towards the liar sentence, only on pain of inconsistency. This is the extended liar paradox, in one of its many guises. Parsons's solution is to distinguish between the linguistic acts of assertion and denial. $\neg T\langle\alpha\rangle$ cannot be asserted, but $T\langle\alpha\rangle$ can be denied.[9]

The distinction between assertion and denial is a venerable and legitimate one, Frege notwithstanding. The trouble with this kind of move, however, is that introducing these auxiliary notions merely invites one to formulate versions of the paradox in terms of them. In 'Assertion, Denial, and the Liar Paradox,' Parsons considers such a possibility. For example, he considers claims such as 'I am not asserting this sentence' and 'I am denying this sentence,' and shows that they do not cause

9 There are, of course, many other suggested solutions. One is to suppose that different tokens of the same type can have different truth values. In the form required here, this has always stuck me as a rather desperate move, which throws spanners into numerous works. It is dealt a damning blow in A. Hazen, 'Contra Buridanum,' *Canadian Journal of Philosophy* **17** (1987) 875-80. Another tack is taken in Smiley's 'True Contradictions, I,' and criticized in my 'True Contradictions, II,' *Proceedings of the Aristotelian Society, Supplementary Volume* **68** (1993) 17-54. A full discussion of all the moves suggested is too long to be undertaken here.

problems. However, there are others. Consider the claim β: it is irrational to assert <β>. Suppose that someone asserted <β>. They would then be asserting something, and at the same time asserting that it is irrational to assert it. This is irrational.[10] Hence asserting <β> is irrational. But this is just <β>, and we have established it. Hence it *is* rational to assert <β>.[11]

We see, then, that explicitly invoking the notions of assertion and denial allows the reformulation of paradox, and so does not solve it.

V Dialetheism and Expressibility

There is more to be said about extended liars, but let us pause to consider Parsons's claim that a glut theorist faces exactly the same problem as a gap theorist here.

A gap theorist, as we have seen, has a problem expressing her views. So does a glut theorist, according to Parsons. Specifically, a glut theorist cannot express the fact he *disagrees* with someone. For example, if you say α, I do not express disagreement when I say $\neg\alpha$. For it is logically possible that both are true. More generally, whatever I say, there are models of both it and α (if only the trivial one where everything is true). As a special case of this, it is claimed, a glut theorist cannot even express the claim that a sentence is true but not false. She can, of course, use these words; but the fact that a sentence is not false does not rule out its being false; it may *still* be both true and false. A similar argument is at the core of the case against 'global paraconsistency' built by Diderick Batens.[12]

The problems for the gap theorist and the glut theorist are not, in fact, exactly the same. The gap theorist has the words to express his views; it is just that these turn out to be untrue, and hence we have a case of self-refutation. The glut theorist has the words to express her views, and

10 Here, I am asserting that something is irrational. One should, presumably, deny the claim that it is rational, too, though this is weaker. It is the stronger position that is warranted since the weaker one is compatible with the person asserting <β> being neither rational nor irrational.

11 This argument is due to G. Littman, 'What Problems does Dialetheism Pose for Rationality?' (Honours Dissertation, University of Queensland 1991). Note that it is not a 'performative paradox,' such as an utterance of the sentence 'I cannot utter this sentence.' The paradox does not depend on anyone actually uttering <β>.

12 'Against Global Paraconsistency,' *Studies in Soviet Thought* 39 (1990) 209-29. See esp. §§ 4 and 5. Parts of the discussion (at least as directed against me) are incorrect, since Batens assumes that falsity entails non-truth. However, the central point survives the correction of this.

these turn out to be true (sometimes they may be false as well, but that's the nature of the beast). Thus, the dialetheist can express truly the view that something is true but not false, in the words I have just used. What he cannot do is ensure that these are consistent (as Batens brings out). The problem is, therefore, that the dialetheist can say nothing that *forces* consistency. But put this way, it is clear that the classical logician cannot do this either. He can say $\neg\alpha$, but this does not rule out a commitment to α: all it does is to ensure that such a commitment occasions a collapse into triviality. But a paraconsistent logician can do this also. Asserting $\neg\alpha$ will not work, but asserting $\alpha \rightarrow F$, where $\rightarrow$ is a detachable conditional and F is 'everything is true,'[13] will do the job.

It may fairly be replied that this still does not express *disagreement*. After all, it may be uttered by someone who thinks that everything *is* true, and so who agrees with α. If we are searching for a mode of expressing disagreement that even this creature can use, then there is, indeed, nothing that can be asserted that will do the trick. But the distinction between assertion and denial makes just as much sense for a glut theorist as it does for a gap theorist. Hence a dialetheist may simply take over Parsons's distinction and use denial to express disagreement.[14]

Does this open the glut theorist to the same objection that I urged against the gap theorist at the end of the last section? No. That argument showed that it is both rational and irrational to assert $<\beta>$. This is a problem for the gap theorist; but for a glut theorist, it is just another contradiction — and one, moreover, of just the kind that we should expect to arise when self-reference and semantic/intensional notions become entangled.[15] It does raise a *practical* problem of course: whether to assert β. One will be (rationally) damned if one does, and damned if one doesn't. Such, unfortunately, may be life.[16]

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13 See *IC*, 8.5.

14 The possibility of this is hinted at in *IC*, 123, n. 10. It is elaborated in 'True Contradictions, II.'

15 On the intensional paradoxes, see my 'Intensional Paradoxes,' *Notre Dame Journal of Formal Logic* **32** 193-211.

16 See, further, *IC* 13.1, 13.2, and 13.5. The main notion of obligation discussed there is legal obligation. However, as I point out, there is no reason to suppose that there is anything special about legal obligation in this regard. Rational obligation, it appears, may well be the same.

VI Exclusion Negation

Let us now return to the extended liar paradox. As I observed in section IV, a gap solution to the liar depends on negation mapping a gap to a gap. This means that further extended paradoxes can be constructed using negation-like functors for which a gap is not a fixed point, as Parsons observes. To use Parsons's own example (358), suppose we define an operator '-' by the following truth conditions:[17]

> $<-\alpha>$ is true (only) iff
> $<\alpha>$ is false (only) or (neither true nor false)
> $<-\alpha>$ is false (only) iff
> $<\alpha>$ is true (only) or (true and false).

Gap theorists may consider the second disjunct in the falsity conditions vacuous, but this is irrelevant for the present. Given that all sentences are true (only) or false (only) or neither (or maybe both), these truth conditions verify the LEM for '-': $-\alpha \vee \alpha$.

Now consider the -liar, a sentence β of the form: $-T<\beta>$. The T-schema tells us that:

$$T<\beta> \text{ iff } -T<\beta>$$

and thus by the LEM and *modus ponens*: $T<\beta> \wedge -T<\beta>$. This is impossible, as the truth conditions for '-' show.

Parsons proposes a solution to this problem, also. Stripped of its complications, the solution is simply that:

(*) if $<\alpha>$ is neither true nor false then both $<<\alpha>$ is true$>$ and $<<\alpha>$ is false$>$ are neither true nor false.

Hence, if $<\alpha>$ is a gap, so is the following sentence: $<\alpha>$ is true or $<\alpha>$ is false or $<\alpha>$ is neither. Hence, we cannot infer the LEM for '-' and the argument breaks down.

The problem with this solution is that it undercuts the whole theory of truth value gaps. For example, if $<\alpha>$ is any sentence $<<\alpha>$ is neither true nor false$>$ is either false or neither true nor false (depending on $<\alpha>$).

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17 Parsons writes '-' as a pair of stacked tildes. I have changed this for typographical reasons. Something seems to have gone wrong with Parsons's definition of '-' as a sentential operator since two distinct definitions are given on separate lines. Presumably, a negation sign has been missed out in the second line of the definition. But sorting through the issues this raises merely cloaks the main point of the present discussion, so I have formulated what follows slightly differently from Parsons, but in a way that I feel sure that he would find acceptable.

Hence, assuming standard semantics for quantifiers, the claim that some sentences are neither true nor false ($\exists x(\neg Tx \wedge \neg Fx)$) comes out as untrue (neither true nor false). Similarly (and assuming that the conditional is something like strong Kleene), the explanation (*) itself comes out as neither true nor false. The same is true when Parsons makes claims such as:

(**) if the sentence on line 3 lacks a truth value, so does the (very same) sentence on line 4. (349)

Parsons indicates that he would deploy the distinction between assertion and denial at this point (346; see esp. n. 12). A gap theorist cannot convey what they want with suitable assertions; they can do it with suitable denials. Several things need to be said about this move. The first is that it should be clearly distinguished from the deployment of denial we came across in section IV. There, the only problem that denial had to solve was how to express the status of the liar sentence itself. Now, it is the whole theory of truth value gaps that it must be used to express.

Next, to claim that the theory of truth value gaps is standardly spelled out in terms of denial would be disingenuous. As a matter of fact, it is standardly spelled out by numerous assertions, e.g., to the effect that certain sentences are neither true nor false. Parsons would presumably say that the theory can be reworked in such a way as to avoid this. But this claim appears to be false: it is not uncommon in philosophy to find a suggestion to the effect that some prima facie contentual notion should be cashed out in terms of a speech act (e.g., moral obligation and commanding). Such suggestions always founder on the problem of embedding.[18] The present case is no exception. Consider, for example, the following conditional, which is certainly true on the usual understanding of gap semantics:

(***) if $<\alpha>$ is not true, any conjunction $<\alpha \wedge \beta>$ is not true.

The negations in this cannot be understood as denials since they are not attached to whole utterances (which force operators must be); and understood as negations, the claim is untrue (as we have observed). Ditto, Parsons's own words (**), quoted above.

Next, even if some account could be given which makes these claims true (for example, by using a more unusual conditional connective), crucial inferential connections would still be destroyed. For example, from (***), given an assertion of '$<\alpha>$ is not true' it is legitimate to infer '$<\alpha \wedge \beta>$ is not true.' But given the denial of '$<\alpha>$ is true,' one cannot

18 See, e.g., P. Geach, *Logic Matters* (Oxford: Blackwell 1972), 8.1 and 8.2.

legitimately infer anything (we need an assertion of the antecedent to apply *modus ponens*).

We are therefore forced to conclude that there are formidable problems to reconstructing the original gap theory in an acceptable way. Hence both it and the subsequent moves that depend on it would seem to collapse.

VII Gluts Again

But we have not finished with the issue yet. Parsons argues that the situation posed by the operator '-' is just as damaging for the dialetheist. Why is this? The truth and falsity conditions of '-' make just as much sense for a dialetheist (though a dialetheist may take the second disjunct in the truth conditions to be vacuous). As before, we can establish $T<\beta>\wedge\text{-}T<\beta>$, and so that this is true. But the truth conditions of '-' entail that no conjunction of this form is true: $\neg T<T<\beta>\wedge\text{-}T<\beta>>$. (As may be checked, whatever value α has, $\alpha\wedge\text{-}\alpha$ is not true.)

But this is *not* a problem for dialetheism. It would be if it aspired to consistency, as the gap approach does. But it does not. The whole point of the approach is to accommodate contradictions, not eliminate them. Neither is this ad hoc, since the very rationale of the dialetheic approach to the semantic paradoxes is precisely that contradictions of this kind may be expected to turn up when self-reference and the T-schema interact. It is true that this contradiction affects the 'metatheory' (i.e., the discourse of the solver), but then a central feature of dialetheism is precisely that one should not expect the metatheory to behave any differently, anyway. There is more to be said about this issue. In particular, it might be suggested that if no contradiction of the form $\alpha\wedge\text{-}\alpha$ is true then the inference $\alpha\wedge\text{-}\alpha$ / β is valid, and hence that triviality ensues in this situation. I have discussed and rejected this argument elsewhere,[19] and so will not pursue the matter here.

VIII Agnostaletheism

Let us, finally, turn to the last section of Parsons's paper. Debates between rival theories are well known in the history of science, and indeed, form its life blood. In the period of such a debate it is not uncommon for a compromise theory to appear. For example, during the

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19 See my 'Boolean Negation and All That,' *Journal of Philosophical Logic* **19** (1990) 201-15, esp. § 5. The discussion there is in terms of Boolean negation, which is slightly different from '-'; but in the relevant ways, it is the same.

debate between the Copernicans and the Ptolemaics, Tycho Brahe proposed the compromise scheme according to which the sun circled the earth, but all other planets circled the sun. Similarly, Lorentz's theory of space/time contraction is a compromise between Newtonian Dynamics and Special Relativity.

Why the phenomenon of compromise candidates is a recurrent one in the history of ideas is an interesting question in the sociology of science, and well outside the scope of anything that can be addressed here (though it is worth noting that compromise candidates seem rarely to become accepted). The relevance of the above observation is simply that we have another instance of the phenomenon here. Parsons observes that gap theorists and glut theorists can endorse the same truth tables for the connectives. He therefore proposes a compromise account according to which the third value is something neutral, which might be looked on either as a gap or as a glut. (Shades of wave/particle duality!) He calls this 'agnostaletheism.'

There must be something wrong with agnostaletheism: for the gap and glut solutions to the semantic paradoxes are not inter-translatable, as we have already seen. What, however, *is* wrong with it? The answer is that there is a lot more to the matter than the simple isomorphism of truth tables.[20] The crucial question, as Parsons indicates (352, n. 17), is what we are to say about asserting sentences that take the third value. As we have seen (in §§ III, V, and VII), there are considerations that drive us towards asserting contradictions and other things with this value. As we saw in sections IV and VI, even the pure gap theorist is driven to this conclusion — however reluctantly. Hence, the third value must be designated. If not, *ex contradictione quod libet* would be valid, and triviality would result. More generally, in many-valued logic designation plays the same role in defining validity as truth plays in two-valued logic. Specifically, designated values are the ones we are interesting in asserting, preserving under inference, etc. Hence, assertable sentences ought to be designated. Thus, glut logic is required, to handle the contradictions, not gap logic.[21] Moreover, by the Teleological Account of truth, since these contradictions are correctly assertable, they are true.

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20 Some discussion can be found in my 'Logic of Paradox Revisited,' *Journal of Philosophical Logic* **13** (1984) 153-79, § 2.

21 Thus, pace Parsons (ibid.), the question of designated values *is* a substantive issue.